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The Calculation of End Areas

BY

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A method of calculating end areas without any reduction of elevations or plotting of cross sections, using direct the notes as taken in the field, thus effecting a great saving in time, giving increased accuracy, and at the same time reducing to a minimum the liability to error.

GENERAL PROPOSITION

Let $P_0P_1P_2P_3P_4P_5$, figures 1 and 2, be any polygonal figures and OX, OY any two lines at right angles to one another, and let the distances $a_1a_2a_3$....represent the horizontal distances of the points $P_1P_2P_3$.... from the vertical lines OY, which in the examples given coincide with the sides P_0P_5.

Let $h_0h_1h_2h_3$....be the vertical distances of the points $P_0P_1P_2P_3$.... from the lines OX giving the + sign to the distances below, and the − sign to those distances above OX. Then, the areas of the polygons $P_0P_1P_2P_3P_4P_5$ will be given by taking one half of the algebraic sum of the products found by multiplying consecutively each horizontal distance $a_1a_2a_3$....by the algebraic difference between the ordinates of the points coming immediately after and before such horizontal distance: that is area of the polygon $P_0P_1P_2P_3P_4P_5$ for figure 1 will be

$$\tfrac{1}{2}\{\, a_1(h_2-h_0)+a_2(h_3-h_1)+a_3(h_4-h_2)+a_4(h_5-h_3)\,\}$$

and for figure 2

$$\tfrac{1}{2}\{\, a_1(h_2-h_0)+a_2(-h_3-h_1)+a_3(-h_4-h_2)+a_4(-h_5+h_3)\,\}$$

or $$\tfrac{1}{2}\{\, a_1(h_2-h_0)-a_2(h_3+h_1)-a_3(h_4+h_2)-a_4(h_5-h_3)\,\}$$

The proof of the foregoing is easily seen, for drawing the horizontal dotted lines AP_1, BP_2, CP_3, DP_4 and the diagonal dotted lines BP_1, CP_2, DP_3, then for figure 1

$\tfrac{1}{2}a_1(h_2-h_0)=\tfrac{1}{2}a_1BP_0=\tfrac{1}{2}a_1(BA+AP_0)=\tfrac{1}{2}a_1BA+\tfrac{1}{2}a_1AP_0=$area BP_0P_1

$\tfrac{1}{2}a_2(h_3-h_1=\tfrac{1}{2}a_2CA=\tfrac{1}{2}a_2(CB+BA)=\tfrac{1}{2}a_2CB+\tfrac{1}{2}a_2BA=$area CBP_1P_2

$\tfrac{1}{2}a_3(h_4-h_2)=\tfrac{1}{2}a_3DB=\tfrac{1}{2}a_3(DC+CB)=\tfrac{1}{2}a_3DC+\tfrac{1}{2}a_3CB=$area DCP_2P_3

$\tfrac{1}{2}a_4(h_5-h_3)=\tfrac{1}{2}a_4P_5C=\tfrac{1}{2}a_4(P_5D+DC)=\tfrac{1}{2}a_4P_5D+\tfrac{1}{2}a_4DC=$area $P_5DP_3P_4$ and the sum of these equals the whole area $P_0P_1P_2P_3P_4P_5$.

For figure 2 the proof is similar, for using the minus signs for the ordinates (corresponding to a fill or embankment) then

$\tfrac{1}{2}a_1(h_2-h_0)=\tfrac{1}{2}a_1BP_0=\tfrac{1}{2}a_1(BA+AP_0)=\tfrac{1}{2}a_1BA+\tfrac{1}{2}a_1AP_0=+$ area $AP_0P_1P_2$

$-\tfrac{1}{2}a_2(h_3+h_1)=-\tfrac{1}{2}a_2(CA)=-\tfrac{1}{2}a_2(CB-BA)=-\tfrac{1}{2}a_2CB+\tfrac{1}{2}a_2BA=-$area ACP_2

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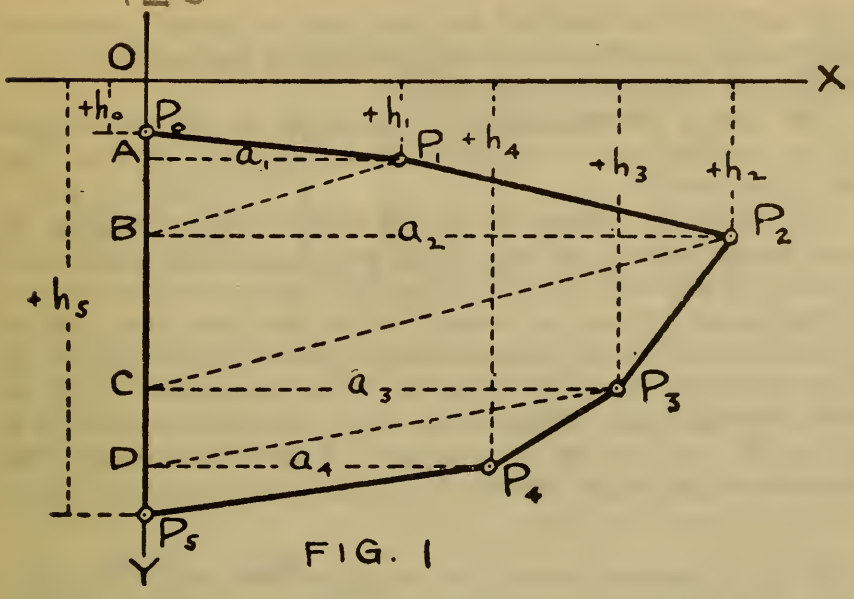

FIG. 1

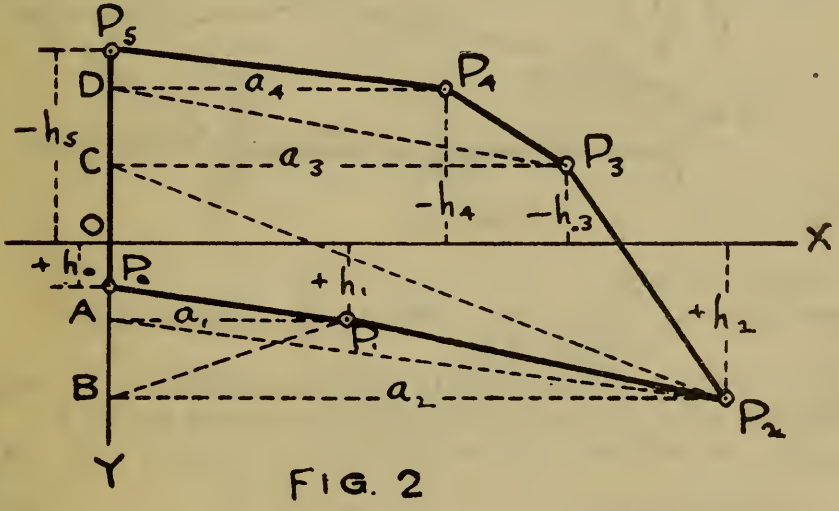

FIG. 2

$-\frac{1}{2}a_3(h_4+h_2) = -\frac{1}{2}a_3BD = -\frac{1}{2}a_3(BC+CD) = -\frac{1}{2}a_3BC$
$-\frac{1}{2}a_3CD = -\text{area } CP_2P_3D$

$-\frac{1}{2}a_4(h_5-h_3) = -\frac{1}{2}a_4CP_5 = -\frac{1}{2}a_4(CD+DP_5) = -\frac{1}{2}a_4CD$
$-\frac{1}{2}a_4DP_5 = -\text{area } DP_3P_4P_5$

and the algebraic sum equals the whole area $P_0P_1P_2P_3P_4P_5$.

To adopt the above conclusions to field work so as to obtain direct from the level notes the areas of the cross sections as taken:—

The horizontal line OX corresponds to the height of instrument and the line OY to a vertical line at centre.

The ordinates $h_0h_1h_2h_3$....are the rod readings taken at various points on the section and the horizontal distances $a_1a_2a_3$....the corresponding distances right or left of the C.L.

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In the field book these are best noted in the form of a fraction as shown on the sample page, the numerator being the rod reading, the denominator the corresponding distance:

The areas to the right and left of the centre line are calculated (as they are so taken in the field) separately and their sum gives the total area of the section.

The horizontal distances to the left could of course be given the minus sign and exactly the same final result would be obtained, but in practice it is simpler to imagine the left half section as being drawn on the transparent left page of a book, so that when this page is turned over the left half section would be superimposed upon the right on the right hand page below, the centre line being represented by the line where the pages join. Thus all the horizontal distances, whether to right or left, can be given the plus sign. Taking the example at the head of the sample page, this section, if plotted, would appear as under.

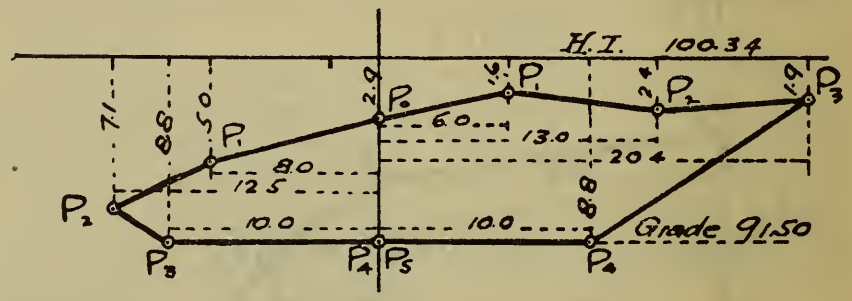

FIG 3

By the formula

area right $= \frac{1}{2}\{ 6.0(2.4-2.9)+13.0(1.9-1.6)+20.4(8.8-2.4)$
$+10.0(8.8-1.9)\}$
$= \frac{1}{2}\{ -6.0,0.5+13.0,0.3+20.4,6.4+10.0,6.9)\ldots\ldots = 100.23$

area left $= \frac{1}{2}\{ 8.0(7.1-2.9)+12.5(8.8-5.0)+10.0(8.8-7.1)\}$
$= \frac{1}{2}(8.0,4.2+12.5,3.8+10.0,1.7)\ldots\ldots\ldots\ldots\ldots = 49.05$

$\therefore$ Total End area $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ 149.28

In working out these areas in the field book the rod readings or ordinates can usually be subtracted mentally and their difference at once jotted down (with the proper sign + or − affixed), as shown on the marginal page.

Should it be found impossible to complete the section with a single height of instrument, proceed as follows: suppose a section on side hill. (See P. 5)

Then, if there be but one or two sections requiring a change in the height of instrument, it would only be necessary after setting up, to find on the upper side the difference 3.2 between 103.54 the new H.I. and 100.34 the original height, for then the rod readings as taken, subtracted from 3.2, would give the proper readings −1.4 and −2.2 referred to the original H.I.

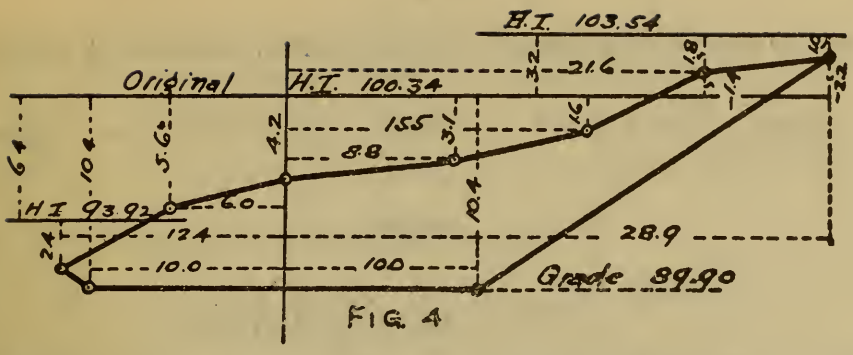

FIG. 4

On the lower or left hand side of the section the new H.I. 93.92 being lower than 100.34, the original height, their difference 6.4 must be added to 2.4, the rod reading as taken, giving for this left side referred to the original H.I. a reading of 8.8.

If, however, there are a number of consecutive sections requiring various changes as above, it would be better perhaps to enter on another page of the level book, under their respective H.I., the rod readings as taken, and then on the marginal page before using such for calculating the end areas reduce them as shown above to the original H.I. by adding or subtracting the constant difference.

The original sections could then be completed by bringing back these corrected readings and the section for such a cross section as that illustrated above would appear in the level book as under.

Station	C. or F.	B.S.	H.I.	F.S.	Elevation	Grade
			100.34			
	R	+12.6	3.1 1.6 / 8.8 15.5	See page..	−1.4 / 21.6	(−2.2 / 28.9)
40		+ 6.2	4.2 / 0			89.90
	L	+ 1.6	5.6 / 6.0	See page..	(8.8 / 12.4)	

The resulting end area would then be as already shown:

area R = ½ { 8.8(1.6−4.2)+15.5(−1.4−3.1)+21.6(−2.2−1.6)
+28.9(10.4+1.4)+10.0(10.4+2.2) }
= ½ { −8.8,2.6−15.5,4.5−21.6,3.8+28.9,11.8+10.0,12.6 } = 146.15

area L = ½ { 6.0(8.8−4.2)+12.4(10.4−5.6)+10.0(10.4−8.8) }
= ½ { 6.0,4.6+12.4,4.8+10.0,1.6 } ..... = 51.56

∴ Total End Area 197.71

Sample page showing manner of keeping the field notes and the method

Station		C. or F.	B.S.	H.I.	F.S.	Elevation	Grade
				100.34			
	R	+6.9		1.6 2.4 (1.9) / 6.0 13.0 (20.4)			
41		+5.9	2.9 / 0				91.50
	L	+1.7		5.0 (7.1) / 8.0 (12.5)			
	R	+3.7		5.5 (5.3) / 8.0 (15.5)			
41+40		+2.9	6.1 / 0				91.30
	L	+0.7		6.7 (8.3) / 6.0 (11.0)			
41+62		0.0		Grade			91.19
	R	−1.6		11.7 (10.9) / 7.0 (10.4)			
42		−2.8	12.1 / 0				91.00
	L	−5.4		14.3 (14.7) / 9.0 (16.1)			
T.P.			0.84	89.12	12.06	88.28	
	R	−3.0		2.4 (1.4) / 9.0 (12.5)			
42+60		−4.6	3.0 / 0				90.70
	L	−6.5		3.1 4.9 (4.9) / 5.0 10.0 (17.7)			

of calculating the end areas direct from these notes as taken.

H.I. to Grade	Calculations, Remarks	End Areas	
		Exc'v'n	Emb'km't
	Road bed 20 ft. Exc. 16 ft. Bank		
	$\frac{-6.0,0.5+13.0,0.3+20.4,6.4+10.0,6.9}{2}$		
+8.8		149.3	
	$\frac{+8.0,4.2+12.5,3.8+10.0,1.7}{2}$		
	$\frac{-8.0,0.8+15.5,3.5+10.0,3.7}{2}$		
+9.0		65.2	
	$\frac{+6.0,2.2+11.0,2.3+10.0,0.7}{2}$		
		0.0	0.0
	$\frac{-7.0,1.2-10.4,2.4-8.0,1.6}{2}$		
+9.3			73.2
	$\frac{+9.0,2.6-16.1,5.0-8.0,5.4}{2}$		
	On rock 40 ft. Right of 42		
	$\frac{-9.0,1.6-12.5,4.0-8.0,3.0}{2}$		
−1.6			127.8
	$\frac{+5.0,1.9+10.0,1.8-17.7,6.5-8.0,6.5}{2}$		

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Station	C. or F.	B.S.	H.I.			Grade
			89.12			
T.P.		9.16	97.52	0.76	88.36	
			Grade			
R	+4.0		7.3 / 2.5 4.2 / 6.5	(3.3 / 16.0)		
43+50	−0.4	7.7 / 0				90.25
L	−2.2		8.1 / 6.4 (9.5 / 11.3)			

The section shown above is on side hill, embankment to the left, excavation to the right, the grade point being as shown 2.5 feet to the right. This must be taken into account in making the calculations so as to keep the excavation and embankment areas separate from one another.

Thus, excavation area to the right is given by

$\frac{1}{2}\{ 2.5(4.2-7.3)+6.5(3.3-7.3)+16.0(7.3-4.2)+10.0(7.3-3.3) \}$

$=\frac{1}{2}\{ -2.5,3.1-6.5,4.0+16.0,3.1+10.0,4.0 \}$ $= +27.9$

There would then be to the right in embankment the small area given by $2.5(7.3-7.7)\frac{1}{2}$ $= -0.5$

and this would have to be added to the embankment area to the left given of course by

$\frac{1}{2}\{ +6.4,1.8-11.3,0.8-8.0,2.2 \} = -7.6$ that is a total area in embankment for the section of -8.1

It might however be considered less confusing in such cases to consider the vertical line as being placed at the grade point instead of at the centre of the section and then all the area to the right of this point would be in excavation and all to the left in embankment.

This would simply mean that in applying the method all horizontal distances taken to the right would be reduced by 2.5 feet and all distances to the left increased by the same amount, that is area right would

$=\frac{1}{2}\{ 4.0(3.3-7.3)+13.5(7.3-4.2)+7.5(7.3-3.3) \}$

$=\frac{1}{2}\{ -4.0,4.0+13.5,3.1+7.5,4.0 \}$ =in excavation 27.9

as before, and area left would

$=\frac{1}{2}\{ 2.5(8.1-7.3)+8.9(9.5-7.7)+13.8(7.3-8.1)+10.5(7.3-9.5) \}$

$=\frac{1}{2}\{ +2.5,0.8+8.9,1.8-13.8,0.8-10.5,2.2 \}$ equal as before to a total area in embankment of 8.1

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